Mode-power fluctuations in optical fibers

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The influence of the relative values of mutual modal delay, source coherence time, and signal modulation time on the power fluctuations between the two polarization states of a single-mode optical fiber is investigated.

The evolution of the power in a propagating mode of an optical fiber is conveniently studied in the frame of coupled power theory. This is a statistical approach that is suitable to describe the evolution of $\langle P_m(z, t)\rangle$, that is, of the power carried by the mth mode, averaged over a statistical ensemble of macroscopically similar fibers.[1]

As far as the predictions on a sample fiber of a real-life experiment are concerned, it is obvious that the knowledge of the average value of $P_m(z, t)$ is not significant unless it is possible to show that the fluctuations of $P_m(z, t)$ are small. In fact, it has been proven that for a monochromatic field the fluctuations of $P_m(z)$ around its average value become large, which seems to indicate that the statistical approach fails to provide a confident prediction of the behavior of mode power in a particular fiber of the ensemble. More precisely, the normalized variance

$$\sigma_m{}^2 = (\langle P_m{}^2\rangle - \langle P_m\rangle^2)/\langle P_m\rangle^2 \tag{1}$$

turns out to be 1 for a multimode lossless optical fiber and 1/3 for a two-mode fiber.[1,2] On the other hand, it has been possible to demonstrate, in the opposite case of finite bandwidth $\delta\omega$ of the exciting source, the existence of a characteristic distance beyond which the normalized variance of the energy per mode,

$$I_m(z) = \int_{-\infty}^{+\infty} P_m(z, t)\mathrm{d}t, \tag{2}$$

becomes negligible[3]; this result, however, has *a priori* no particular implication for the time behavior of $P_m(z, t)$, which determines the modal-dispersion characteristics of the fiber.

In this Letter we prove, limiting ourselves for simplicity to the relevant case of a single-mode fiber, that there exist significant situations in which σ_m tends to zero, a circumstance that justifies the validity of the statistical approach in the sense specified above. More explicitly, $\sigma_m \to 0$ whenever the chain of inequalities

$$T_c \ll \tau_{12} \ll T_m \tag{3}$$

holds true between the mutual delay time $\tau_{12} = z|1/V_1 - 1/V_2|$ of the two polarization states, the coherence time $T_c = 2\pi/\delta\omega$ of the source, and the modulation time T_m of the signal. Conversely, no definite information can be obtained on the variance σ_m whenever relation (3) is not satisfied; so in this case, which as an example may occur when one employs a narrow-band semiconductor laser ($T_c \simeq 100$ nsec) modulated in the high-frequency regime ($T_m \simeq 1$ nsec), the statistical approach has to be handled with caution.

We write the transverse part of the electric field traveling in the single-mode fiber as

$$\begin{aligned}\mathbf{E}(\mathbf{r}, z, t) = {}& \mathbf{e}_1(\mathbf{r}) \\ & \times \exp[i\omega_0 t - i\beta_1(\omega_0)z]\Phi_1(z, t) + \mathbf{e}_2(\mathbf{r}) \\ & \times \exp[i\omega_0 t - i\beta_2(\omega_0)z]\Phi_2(z, t),\end{aligned} \tag{4}$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ represent the spatial configuration of the two orthogonally polarized states and Φ_1 and Φ_2 are the relative slowly varying amplitudes, ω_0 being the central frequency of the carrier. The amplitudes Φ_1 and Φ_2 are expressible in terms of the expansion coefficients $c_1(z, \omega)$ and $c_2(z, \omega)$ through the relation

$$\begin{aligned}\Phi_{1,2}(z, t) = {}& \int \mathrm{d}\omega c_{1,2}(z, \omega) \\ & \times \exp\{i(\omega - \omega_0)t - i[\beta_{1,2}(\omega) - \beta_{1,2}(\omega_0)]\},\end{aligned} \tag{5}$$

where c_1 and c_2 obey, in a lossless case, the coupled set of equations

$$\begin{aligned}\mathrm{d}c_1(z, \omega)/\mathrm{d}z &= iK(z) \exp[i\Delta\beta(\omega)z]c_2(z, \omega), \\ \mathrm{d}c_2(z, \omega)/\mathrm{d}z &= iK(z) \exp[-i\Delta\beta(\omega)]c_1(z, \omega),\end{aligned} \tag{6}$$

$K(z)$ being a real random-coupling coefficient associated with fiber imperfections and $\Delta\beta \equiv \beta_1 - \beta_2$. By using Eqs. (5) and (6) it is easy to show that $\Phi_{1,2}$ satisfy the following set of equations:

$$\begin{aligned}\left(\frac{\partial}{\partial z} + \frac{1}{V_1}\frac{\partial}{\partial t}\right)\Phi_1(z, t) &= iK(z) \exp[i\Delta\beta(\omega_0)z]\Phi_2(z, t), \\ \left(\frac{\partial}{\partial z} + \frac{1}{V_2}\frac{\partial}{\partial t}\right)\Phi_2(z, t) &= iK(z) \exp[-i\Delta\beta(\omega_0)z]\Phi_1(z, t),\end{aligned} \tag{7}$$

where we have introduced the group velocity

$$V_i = (\mathrm{d}\beta_i/\mathrm{d}\omega)^{-1}_{\omega=\omega_0}$$

of the ith mode ($i = 1, 2$) and neglected chromatic dis-

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persion. Equations (7) can be formally integrated, and they furnish

$$\Phi_1(z,t) = \Phi_1(z=0, t-z/V_1) + i\int_0^z K(z') \times \exp[i\Delta\beta(\omega_0)z']\Phi_2\left(z', t+\frac{z'-z}{V_1}\right)\mathrm{d}z',$$

$$\Phi_2(z,t) = \Phi_2(z=0, t-z/V_2) + i\int_0^z K(z') \times \exp[-i\Delta\beta(\omega_0)z']\Phi_1\left(z', t+\frac{z'-z}{V_2}\right)\mathrm{d}z', \tag{8}$$

which can be solved by successive iterations, resulting in

$$\Phi_1(z,t) = \Phi_1(z=0, t-z/V_1) + i\int_0^z K(z') \times \exp[i\Delta\beta(\omega_0)z']\Phi_2\left[z=0, t-\frac{z}{V_1} + z'\left(\frac{1}{V_1}-\frac{1}{V_2}\right)\right]\mathrm{d}z' + i^2\int_0^z\int_0^{z'} K(z')K(z'') \times \exp[i\Delta\beta(\omega_0)(z'-z'')]\Phi_1\left[z=0, t-\frac{z}{V_1} + (z'-z'')\left(\frac{1}{V_1}-\frac{1}{V_2}\right)\right]\mathrm{d}z'\mathrm{d}z'' + \ldots, \tag{9}$$

an analogous expression holding true for $\Phi_2(z,t)$. Let us now assume that the boundary condition $\Phi_i(z=0, t)$, where $i = 1, 2$, can be factorized into the product of a slowly varying function $S(t)$ (representing the modulation of the signal) and a rapidly varying random function $F(t)$ (responsible for the source bandwidth) in the form

$$\Phi_i(z=0,t) = \eta_i S(t)F(t), \quad i = 1, 2, \tag{10}$$

where η_i is a constant that takes into account the efficiency of the ith-mode excitation. If the characteristic scale of variation T_m of $S(t)$ is much larger than the mutual delay time τ_{12}, one can rewrite Eq. (9) in the form

$$\Phi_1(z,t) = \eta_1 S(t-z/V_1)F(t-z/V_1) + i\eta_2 S(t-z/V_1)\int_0^z K(z') \times \exp[i\Delta\beta(\omega_0)z']\, F\left[t-\frac{z}{V_1} + z'\left(\frac{1}{V_1}-\frac{1}{V_2}\right)\right]\mathrm{d}z' + i\eta_1 S(t-z/V_1)\int_0^z\int_0^{z'} K(z')K(z'') \times \exp[i\Delta\beta(\omega_0)(z'-z'')] \times F\left[t-\frac{z}{V_1} + (z'-z'')\left(\frac{1}{V_1}-\frac{1}{V_2}\right)\right]\mathrm{d}z'\mathrm{d}z'' + \ldots. \tag{11}$$

If $T_m \gg T_c$, the power per mode $P_m(z,t)$ is proportional to $\overline{|\Phi_m(z,t)|^2}$, where the overbar stands for a time average over an interval long compared with the coherence time T_c of the source but short compared with T_m. We thus obtain (omitting for simplicity a proportionality factor)

$$P_1(z,t) = \overline{|\Phi_1(z,t)|^2} = |\eta_1|^2|S(t-z/V_1)|^2\overline{|F(0)|^2} + |\eta_2|^2|S(t-z/V_1)|^2\int_0^z\int_0^z K(z')K(z'') \times \exp[i\Delta\beta(\omega_0)(z'-z'')] \times \overline{F(0)F^*\left[(z'-z'')\left(\frac{1}{V_2}-\frac{1}{V_1}\right)\right]}\mathrm{d}z'\mathrm{d}z'' - |\eta_1|^2|S(t-z/V_1)|^2\left\{\int_0^z\int_0^{z'} K(z')K(z'') \times \exp[i\Delta\beta(\omega_0)(z'-z'')] \times \overline{F^*(0)F\left[(z'-z'')\left(\frac{1}{V_1}-\frac{1}{V_2}\right)\right]}\mathrm{d}z'\mathrm{d}z'' + \text{c.c.}\right\}\ldots, \tag{12}$$

from which it follows that

$$P_1(z,t) = \frac{|S(t-z/V_1)|^2}{\int_{-\infty}^{+\infty}|S(t-z/V_1)|^2\mathrm{d}t} I_1(z), \tag{13}$$

where $I_1(z)$ represents the total energy in mode 1 crossing the fiber at section z [see Eq. (2)]. Therefore $P_1(z,t)$, as a random function of the coupling coefficient $K(z)$, has the same statistical behavior as $I_1(z)$ so that, according to the results of Ref. 3, its normalized variance tends to vanish for $\tau_{12} > T_c$. This in turn implies that the description of $P_1(z,t)$, as obtained in the frame of the statistical coupled power theory, can be assumed to represent to a good approximation the actual behavior of $P_1(z,t)$ on a particular fiber of the ensemble. As the above derivation shows, the result holds true whenever inequality (3) is satisfied, while no definite conclusions can be drawn when it is not valid.

We note finally that, although the analytic treatment of this Letter concerns a single-mode fiber, the main conclusions also apply to a multimode fiber. In this case, keeping in mind that the exchange of power takes place in practice only among adjacent groups of modes (see, e.g., Ref. 1, Sec. 5.4), inequality (3) has to be satisfied by taking for τ_{12} the mutual delay between any two groups of modes. This condition indeed seems to be fulfilled for the experimental situation concerning the measurement of the dependence of modal dispersion on the traveled distance z in a multimode fiber.[4] In fact, the observed square-root-of-z dependence of pulse broadening is consistent with the theoretical prediction of the statistical approach.[1,5]

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